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HEAT TRANSFER FOR A FREELY FLOWING FILM

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A numerical method has been applied to a turbulent-transport model to examine the cooling of a liquid film in a circular tube.

Most previous studies of film processes have been concerned with the heating of films or with heat transfer involving phase transitions [1-8], but the methods are limited in application to various flow conditions; e.g., laminar flow was envisaged in [4], while wave or turbulent flow was considered in [5-8].

We have made a numerical study of the cooling of a freely flowing convective film of liquid within a vertical tube with laminar and turbulent modes of flow; this is of some practical interest, since film condensation in power systems usually requires supercooling of the liquid in order to ensure normal pump operation. Turbulent transport is assumed, and satisfactory results are obtained for the entire flow range.

The steady-state axisymmetric free flow of the film on the internal surface of a vertical tube is considered subject to the condition that the mass flow rate and the physical parameters of the liquid are constant, while the longitudinal pressure gradient is zero. The following are the differential equations for conservation of momentum and energy:

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = \frac{1}{r} \frac{\partial}{\partial y} \left(r\mu_{\text{eff}} \frac{\partial u}{\partial y} \right) + g(\rho_L - \rho_G),$$
$$\rho u \frac{\partial H}{\partial x} + \rho v \frac{\partial H}{\partial y} = \frac{1}{r} \frac{\partial}{\partial y} \left[r \left(\frac{\mu_{\text{eff}}}{\text{Pr}_{\text{eff}}} \frac{\partial H}{\partial y} + \mu_{\text{eff}} u \frac{\partial u}{\partial y} \right) \right]. \tag{1}$$

The total enthalpy is

$$H = c_p (T' - T) + \frac{u^2}{2}, \tag{2}$$

where T' is the inlet liquid temperature.

The effective viscosity and Prandtl number are given by:

$$\mu_{\text{eff}} = \mu + \mu_t, \quad \text{Pr}_{\text{eff}} = \frac{\mu_{\text{eff}}}{\dfrac{\mu}{\text{Pr}} + \dfrac{\mu_t}{\text{Pr}_t}}. \tag{3}$$

The value of Pr_t is taken as constant at 0.9 throughout the layer.

The turbulent viscosity is derived from Prandtl's mixing-length hypothesis:

$$\mu_t = \rho l^2 \left| \frac{\partial u}{\partial y} \right|. \tag{4}$$

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This length was calculated for van Drist's model as modified for Re $\leqslant 10^4$ [9]:

$$l = 0.4y' \left\{1 - \exp\left[-\frac{y'\sqrt{\tau_w \rho}}{26\mu}\left(\frac{\tau}{\tau_w}\right)^2\right]\right\}. \tag{5}$$

If necessary, the roughness can be incorporated by superposition, with the surface replaced by a hypothetical surface having a finite mixing length [10].

The boundary conditions for momentum conservation are

$$u_w = 0, \; \left.\frac{\partial u}{\partial y}\right|_I = 0. \tag{6}$$

The energy equation for the boundary layer was solved subject to the following boundary conditions:

$$\text{I. } T_w = \text{const}, \; T_I = T_s, \tag{7}$$

$$\text{II. } q_w = \alpha_c (T_w - \bar{T}_c), \; T_I = T_s, \tag{8}$$

$$\text{III. } q_w = \alpha_c (T_w - \bar{T}_c), \; \left.\frac{\partial T}{\partial y}\right|_I = 0. \tag{9}$$

The mass flow rate and temperature at the inlet were specified as corresponding to saturation at the given pressure; the film thickness was determined from the equality between the force of gravity and the tangential stress at the wall.

The Patankar–Spalding method [11] was used; the following numerical results are for free flow of a film of N_2O_4. The physical parameters were taken from [12]. The following parameters were used: $P = 2.2 \cdot 10^5$ Pa, $\dot{m} = 0.01$ kg/sec, $R = 10^{-2}$ m, Pr = 4.2. The numerical solutions were obtained with a Minsk-32 computer. The mean run time was 5 min for a channel length (film range) of 1 m.

Figure 1a compares the mean film thickness found from the solution of the boundary-layer equations with other sources of data; the turbulent-transport model automatically incorporates the transition from laminar flow to turbulent. The transfer occurs over the range Re = 1200-2400, which is in agreement with the results of [13], where similar relationships were given for the tangential stress.

It is stated [14] that surface tension damps the fluctuations in the free surface in a turbulent flow; Fig. 2 shows that our turbulence model describes this phenomenon correctly.

The surface curvature has a marked effect on the film thickness for $\delta/R > 0.1$ (Table 1). This result is in agreement with published data [15, 16].

Figure 1b compares the heat-transfer results subject to the boundary conditions of (7) with equations that have been fitted to measurements on the heating of films flowing on the

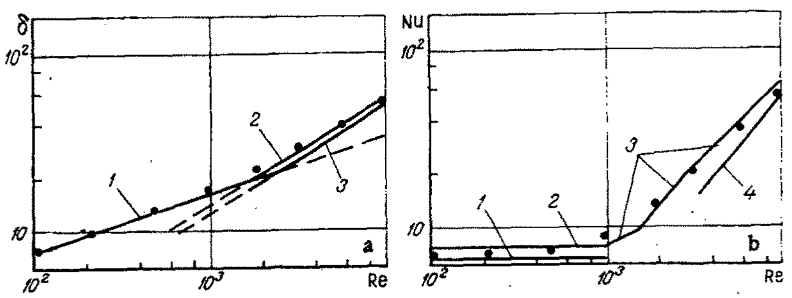

Fig. 1

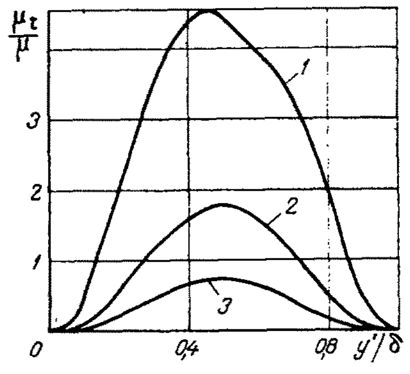

Fig. 2

Fig. 1. Variations in film thickness (a) and Nusselt number (b); the points are from our method; a: 1) Nusselt; 2) [2]; 3) [15]; b: 1) Nusselt; 2) [16]; 3) [17]; 4) [8]; δ, 10^{-5} m.

Fig. 2. Distribution of the turbulent viscosity over the film thickness: 1) Re = 5614; 2) 3089; 3) 1872.

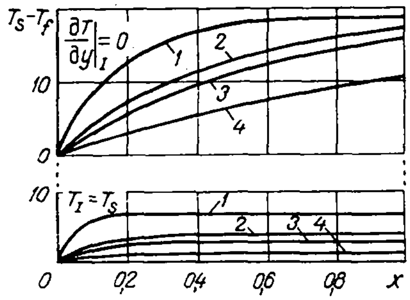

Fig. 3

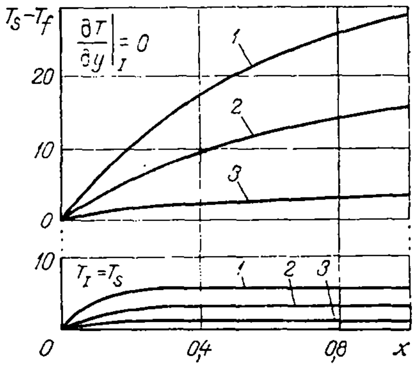

Fig. 4

Fig. 3. Effects of heat-transfer coefficient on film supercooling (T_c = 293°K, Re = 1872): 1) α = 5000 W/m^2 · deg K; 2) 1000; 3) 700; 4) 300; x, m; $T_s - T_f$, °K.

Fig. 4. Effects of cooling-medium temperature on film supercooling (α = 700 W/m^2 · deg K, Re = 1872): 1) T_c = 278°K; 2) 293; 3) 308; x, m; $T_s - T_f$, °K.

TABLE 1. Effects of Surface Curvature on Film Thickness (Re = 1872)

R, 10^{-3} m	1,00	2,25	5,00	10,00	20,00
δ, 10^{-4} m	2,150	2,187	2,199	2,202	2,204
δ/R	0,215	0,097	0,044	0,022	0,011

outer surfaces of tubes. This comparison is permissible because the surface curvature does not affect the behavior for the given value of δ/R, while the sense of the heat flux is without effect if the physical parameters are constant. The heat-transfer coefficient from the film is

$$\alpha_f = \frac{q}{\bar{T}_f - T_w}. \quad (10)$$

When waves are present, the heat transfer increases with Re from the value obtained by Kapitsa [16] (which itself is 21% above Nusselt's value), where the limiting value for laminar flow is:

$$\mathrm{Nu} = 6.4. \quad (11)$$

This variation in Nu (for given boundary conditions) is due to gradual damping of the turbulent pulsations.

We also calculated the heat-transfer coefficient for laminar flow in the form $\alpha_f = q/(T_s - T_w)$; in that case, Nu was taken as 3.97, which corresponds to heat transfer by conduction alone ($\alpha = \lambda/\delta$).

These model results allow one to evaluate the various cooling conditions; Figs. 3 and 4 show results obtained with the boundary conditions of (8) and (9). The cooling of the film increases with the heat-transfer coefficient of the cooling medium and as the temperature decreases. If the condition $\partial T/\partial y = 0$ is met at the free surface of the film, the film temperature falls throughout the flow length (pipe length). The temperature reduction is largest and the film temperature approaches that of the cooling medium.

These results show that the attainable supercooling is only slight if the free surface is in contact with the vapor phase ($T_I = T_s$), and this occurs over the initial length of about 0.2-0.3 m; larger degrees of supercooling require adiabatic parts on the hot side adjacent to the free surface of the film.

NOTATION

c_p, specific heat at constant pressure, g, gravitational acceleration; H, enthalpy; l, mixing length; $\dot{m}$, mass flow rate; Nu = $4\delta a/\lambda$, Nusselt number; P, pressure; Pr = $\mu c_p/\lambda$, Prandtl number; q, heat flux density; R, tube radius; Re = $4\dot{m}/2\pi R\mu$, Reynolds number; r, radial coordinate; u, velocity component in the x direction; v, velocity component in the y direction; x, distance along the axis; y, distance from the free surface of film; y', distance from wall; α, heat-transfer coefficient; δ, film thickness; λ, thermal conductivity; μ, dynamic viscosity; ρ, density; η, shear stress. Subscripts: c, cooling medium; eff, effective properties; f, film; G, gas; I, free film surface; L, liquid; s, saturation line; t, turbulent component; w, wall.

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INFLUENCE OF VARIABLE VISCOSITY ON THE HEAT TRANSFER IN A LAMINAR FLUID FILM

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An expression taking account of the heat flux direction during heat transfer in a laminar fluid film is obtained from the approximate solution of the equations of motion and heat conduction.

The influence of a temperature change in the viscosity across the layer is not taken into account in the majority of papers [1-4] examining heat transfer in fluid films although experimental results on the cooling, heating, and also the change in the temperature drop yield different results. Some authors [5] use the factor $(\nu_f/\nu_w)^{0.25}$ by analogy with heat transfer in pipes [6] or on the basis of the Bays experiments [7] conducted in short tubes.

We attempted to estimate the influence of variable viscosity on heat transfer on the basis of an analytical solution of the fundamental equations.

Stable two-dimensional flow of a laminar fluid layer along a vertical wall with a semi-infinite heating section is considered. The Ox axis of the coordinate system is on the solid boundary in the flow direction, while the Oy axis is perpendicular to the stream and the wall. For $x < 0$ the wall temperature is t_0, while for $x \geqslant 0$ it is given by a smooth function $t_w(x)$. The change in the kinetic viscosity coefficient $\nu(t)$ is approximated by a hyperbola. Heat

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